

Experimental test of rotational invariance and entanglement of photonic six-qubit singlet state

Magnus Rådmark¹, Marek Żukowski² and Mohamed Bourennane¹

¹*Physics Department, Stockholm University, SE-10691 Stockholm, Sweden*

²*Institute for Theoretical Physics and Astrophysics,
Uniwersytet Gdański, PL-80-952 Gdańsk, Poland*

(Dated: March 18, 2009)

We experimentally test invariance properties of the six-photonic-qubits generalization of a singlet state. Our results clearly corroborate with theory. The invariance properties are useful to beat some types of decoherence. We also experimentally detect entanglement in the state using an appropriate witness observable, composed of local observables. Our results clearly indicate that the tested setup, which is very stable, is a good candidate for realization of various multi-party quantum communication protocols. The estimated fidelity of the produced state is very high: it reveals very strong EPR six-photon correlations of error rate below 6%.

PACS numbers: 03.67.-a, 03.67.Mn, 03.67.Pp.

It is well known that quantum information processing relies on preparation, manipulation, and detection of superpositions of quantum states. Superpositions, however, are very fragile and are easily destroyed by the decoherence processes due to unwanted couplings with the environment [2]. Such uncontrollable influences cause noise in the communication, or errors in the outcome of a computation. Several strategies have been devised to cope with decoherence. For instance, if the qubit-environment interaction, no matter how strong, exhibits some symmetry, then there exist quantum states which are invariant under this interaction. These states are called decoherence-free (DF) states, and allow to protect quantum information [3, 4]. This situation occurs for instance when the spatial (temporal) separation between the carriers of the qubits is small relative to the correlation length (time) of the environment. Experimental efforts investigating features of DF systems have been carried out to demonstrate the properties of a specific two-qubit DF state [5], and the existence of three-qubit noiseless subsystems [6]. Bourennane *et al.* [7] produced the four-photon polarization-entangled state which is a generalization of the singlet, $|\Psi_4^-\rangle$, and demonstrated its invariance under general collective noise and experimentally showed the immunity of a qubit encoded in this state.

To encode an arbitrary two-qubit state Cabello has theoretically constructed DF states formed by six qubits. One of these states is $|\Psi_6^-\rangle$ [8]. It is invariant under transformations which consist of identical unitary transformations of each individual constituent[3]:

$$U^{\otimes 6}|\Psi_6^-\rangle = |\Psi_6^-\rangle, \quad (1)$$

where $U^{\otimes 6} = U \otimes \dots \otimes U$ denotes the tensor product of six identical unitary operators U . Besides protecting against collective noise, the DF states are useful for communication of quantum information between two observers who do not share a common reference frame [9]. In such a sce-

nario, any realignment of the receiver's reference frame corresponds to an application of the same transformation to each of the qubits which were sent. The states $|\Psi_6^-\rangle$ can also be used for secure quantum multiparty cryptographic protocols such as the six-party secret sharing protocol [10, 11].

Recently multiphoton interferometry based on parametric down conversion reached the stage at which one can observe genuine six-photon interference. The experiment of ref. [12] a generalization of the schemes suggested in [13] was used. In our recent experiment [15] we used a generalization of the blueprint of [16], and its realization [17]. We obtained a six-photon invariant entangled state by pulse pumping just *one crystal* and extracting the third order process. This is done only via suitable filtering, and the interference is observed behind four beamsplitters. The setup is strongly robust, as it faces no alignment problems. The observed six-photon correlation with high fidelity agree with the ones of the theoretical $|\Psi_6^-\rangle$.

In this paper present results of the invariance tests of the experimental correlations attributable to $|\Psi_6^-\rangle$. This is done by sequence measurements of three mutually complementary polarizations at all six detection stations (linear vertical/horizontal, linear diagonal/antidiagonal, circular right/left). The other interesting feature of the state is that it reveals various interesting types of entanglement within the subsystems. This is studied here theoretically, and compared with the data. Finally we present tests aimed at verification of the entanglement of the obtained state. We use the toolbox of entanglement witnesses provided in [20].

The state component corresponding to the emission in a PDC process of six photons into two spatial modes in a PDC process is proportional to

$$(a_{0H}^\dagger b_{0V}^\dagger + e^{i\phi} a_{0V}^\dagger b_{0H}^\dagger)^3 |0\rangle. \quad (2)$$

where a_{0H}^\dagger (b_{0V}^\dagger) is the creation operator for one hori-

horizontal (vertical) photon in mode a_0 (b_0), and conversely; C is a normalization constant, α is a function of pump power, non-linearity and length of the crystal, ϕ is the phase difference between horizontal and vertical polarizations due to birefringence in the crystal, and $|0\rangle$ denotes the vacuum state. This is a good description of the initial six-photon state, provided one collects the photons under conditions that allow the indistinguishability between separate two-photon emissions [14]. A particle interpretation of this term can be obtained through its expansion

$$(a_{0H}^{\dagger 3} b_{0V}^{\dagger 3} + 3e^{i\phi} a_{0H}^{\dagger 2} b_{0V}^{\dagger 2} a_{0V}^{\dagger} b_{0H}^{\dagger} + 3e^{2i\phi} a_{0H}^{\dagger} b_{0V}^{\dagger} a_{0V}^{\dagger 2} b_{0H}^{\dagger 2} + e^{3i\phi} a_{0V}^{\dagger 3} b_{0H}^{\dagger 3})|0\rangle, \quad (3)$$

and is given by the following superposition of photon number states:

$$|3H_{a_0}, 3V_{b_0}\rangle + e^{i\phi}|2H_{a_0}, 1V_{a_0}, 2V_{b_0}, 1H_{b_0}\rangle + e^{2i\phi}|1H_{a_0}, 2V_{a_0}, 1V_{b_0}, 2H_{b_0}\rangle + e^{3i\phi}|3V_{a_0}, 3H_{b_0}\rangle, \quad (4)$$

where e.g. $3H_{a_0}$ denotes three horizontally polarized photons in mode a_0 . The third order PDC is fundamentally and intrinsically different than a product of three entangled pairs. Due to the bosonic nature of photons the emissions of completely indistinguishable photons are favored compared with the ones with orthogonal polarization.

We report experimental observations which are aimed at testing whether the correlations produced in our setup are indeed rotationally invariant. The invariant six-qubit polarization entangled state given by the following superposition of a six-qubit Greenberger-Horne-Zeilinger (GHZ) state and two products of three-qubit W states.

$$|\Psi_6^-\rangle = \frac{1}{\sqrt{2}}|GHZ_6^-\rangle + \frac{1}{2}(|\overline{W}_3\rangle|W_3\rangle - |W_3\rangle|\overline{W}_3\rangle). \quad (5)$$

The GHZ state is here defined as $|GHZ_6^-\rangle = \frac{1}{\sqrt{2}}(|HHHVVV\rangle - |VVVHHH\rangle)$, and the W-state is defined as $|W_3\rangle = \frac{1}{\sqrt{3}}(|HHV\rangle + |HVV\rangle + |VHH\rangle)$. $|\overline{W}\rangle$ is the spin-flipped $|W\rangle$, and H and V denote horizontal and vertical polarization, respectively. This state is obtained from the third order emission of the PDC process eq. (2) with the phase $\phi = \pi$. The emitted photons are beam-split into six modes and one selects the terms with one photon in each mode.

It is easy to see that if one moves into the spin description of the polarization variables, the state is a singlet (total spin equal to zero) of a composite system consisting of six spins $1/2$.

In our experiment we use a frequency-doubled Ti:Sapphire laser (80 Mhz repetition rate, 140 fs pulse length) yielding UV pulses with a central wavelength at 390 nm and an average power of 1300 mW. The pump beam is focused to a 160 μm waist in a 2 mm thick BBO

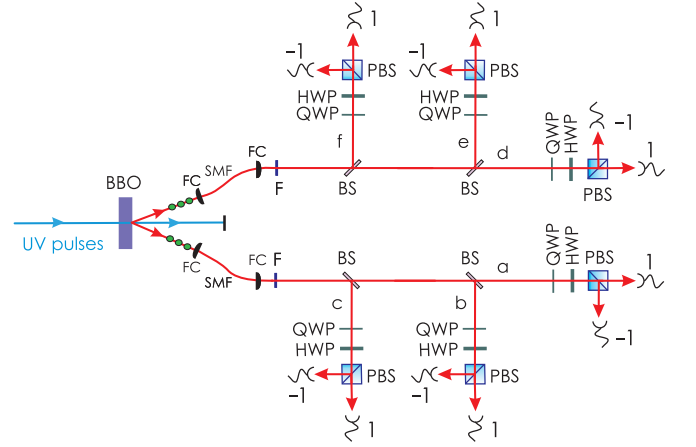


FIG. 1: Experimental setup for generating and analyzing the six-photon polarization-entangled state. The six photons are created in third order PDC processes in a 2 mm thick BBO pumped by UV pulses. The intersections of the two cones obtained in non-collinear type-II PDC are coupled to single mode fibers (SMF) wound in polarization controllers. Narrow band interference filters (F) ($\Delta\lambda = 3$ nm) serve to remove spectral distinguishability. The coupled spatial modes are divided into three modes each by 50%-50% beam splitters (BS). Each mode can be analyzed in arbitrary basis using half- and quarter wave plates (HWP and QWP) and a polarizing beam splitter (PBS). Simultaneous detection of six photons (two single photon detectors for each mode) are being recorded by a twelve channel coincidence counter.

(β -barium borate) crystal. Half wave plates and two 1 mm thick BBO crystals are used for compensation of longitudinal and transversal walk-offs. The third order emission of non-collinear type-II PDC is then coupled to single mode fibers (SMF), defining the two spatial modes at the crossings of the two frequency degenerated down-conversion cones. Leaving the fibers the down-conversion light passes narrow band ($\Delta\lambda = 3$ nm) interference filters (F) and is split into six spatial modes (a, b, c, d, e, f) by ordinary 50%–50% beam splitters (BS), followed by birefringent optics to compensate phase shifts in the BS's. Due to the short pulses, narrow band filters, and single mode fibers the down-converted photons are temporally, spectrally, and spatially indistinguishable [14], see Fig. 1. The polarization is being kept by passive fiber polarization controllers. Polarization analysis is implemented by a half wave plate (HWP), a quarter wave plate (QWP), and a polarizing beam splitter (PBS) in each mode. The outputs of the PBS's are lead to single photon silicon avalanche photo diodes (APD) through multi mode fibers. The APD's electronic responses, following photo detections, are being counted by a multi channel coincidence counter with a 3.3 ns time window. The coincidence counter registers any coincidence event between the 12 APD's as well as single detection events.

Fig. 2a shows the probabilities to obtain each of the 64

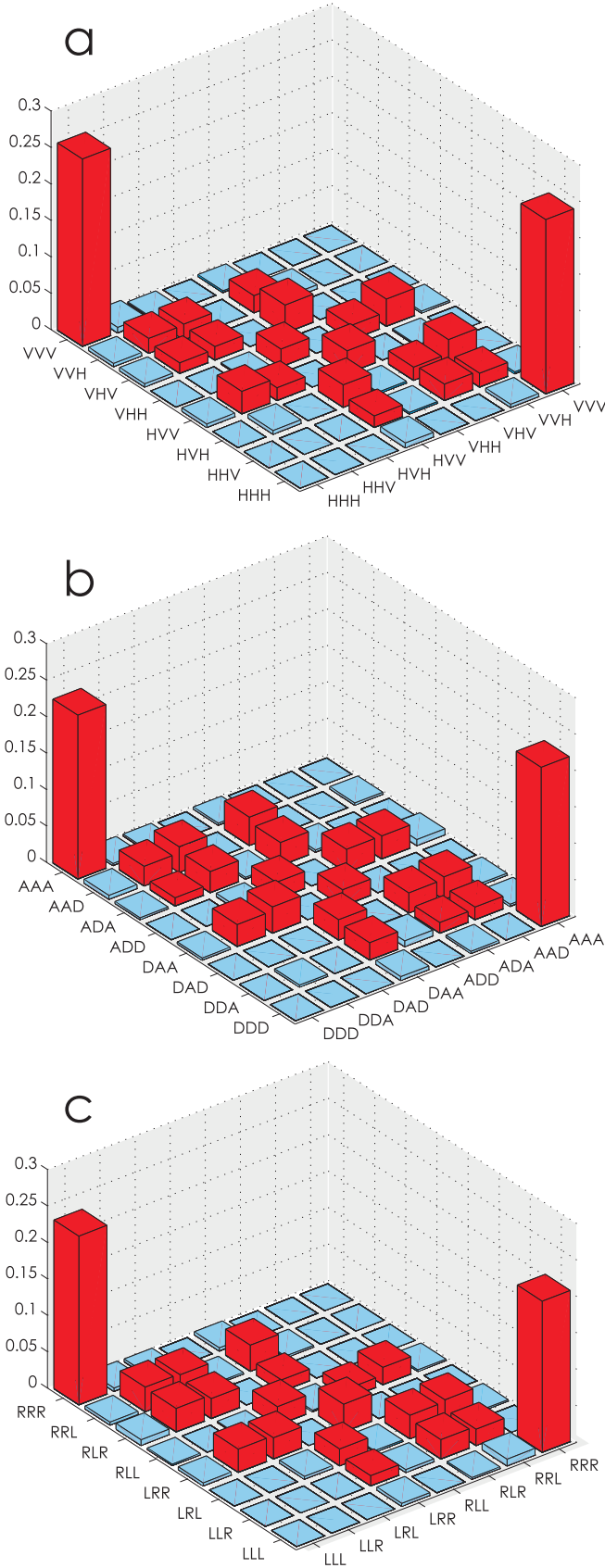


FIG. 2: Experimental results of the six-photon invariant state. Six-fold coincidence probabilities corresponding to detections of one photon in each mode in H/V -basis (a), D/A -basis (b), and L/R -basis (c). The values of the correlation functions are $-87.88\% \pm 4.46\%$, $-87.59\% \pm 4.97\%$, and $-86.79\% \pm 4.27\%$ respectively. Comparing the three measurement results makes the invariance of our state obvious. For the pure $|\Psi_6^-\rangle$ state the light blue bars would be zero and in our experiment their amplitudes are all in the order of the noise. The measurement

possible sixfold coincidences with one photon detection in each spatial mode, measuring all qubits in $\{|H\rangle, |V\rangle\}$ basis. The peaks are in very good agreement with theory: half of the detected sixfold coincidences are to be found as $HHHVVV$ and $VVVHHH$, and the other half should be evenly distributed among the remaining events with three H and three V detections. This is a clear effect of the bosonic interference (stimulated emission) in the BBO crystal giving higher probabilities for emission of indistinguishable photons.

The six-photon state $|\Psi_6^-\rangle$ is invariant under identical (unitary) transformations U in each mode. Experimentally this can be shown by using *identical* settings of all polarization analyzers: no matter what the setting is, the results should be similar. Our results for measurements in $\{|D\rangle, |A\rangle\}$ (Diagonal/Antidiagonal, $|D/A\rangle = (|H\rangle \pm |V\rangle)/\sqrt{2}$) and $\{|L\rangle, |R\rangle\}$ (Left/Right, $|L/R\rangle = (|H\rangle \pm i|V\rangle)/\sqrt{2}$) polarization bases are presented in fig. 2.b and 2.c. The invariance of the probabilities with respect the joint changes of the measurement basis in all modes is clearly visible. We clearly observe, in the results of these three different settings measurements, the small and uniform noise contribution.

Another property of the $|\Psi_6^-\rangle$ is that it exhibit perfect EPR correlations between measurement results in different modes. We obtain the corrections $\langle \Psi_6^- | \sigma_z^{\otimes 6} | \Psi_6^- \rangle = -0.879 \pm 0.045$, $\langle \Psi_6^- | \sigma_x^{\otimes 6} | \Psi_6^- \rangle = -0.876 \pm 0.050$, and $\langle \Psi_6^- | \sigma_y^{\otimes 6} | \Psi_6^- \rangle = -0.868 \pm 0.043$. which are close the theoretical value of -1 . From these results and the approximation that our noise to white noise we have estimate the fidelity $F = \langle \Psi_6^- | \rho_{exp} | \Psi_6^- \rangle = 0.876 \pm 0.045$ where ρ_{exp} is the experimental the six-photon density. The estimated fidelity clearly shows that the setup is able to produce correlations due to six photon entangled states with unprecedented precision (error rate below 6%).

Conditioning on a detection of one photon in a specific state we have also obtained four different five-photon entangled states. In the computational basis the projection of the last qubit on $|V\rangle$ leads to

$${}_f\langle V | \Psi_6^- \rangle = \frac{1}{\sqrt{2}} |HHHV\rangle - \frac{1}{\sqrt{3}} |W_3\rangle | \Psi_2^+ \rangle + \frac{1}{\sqrt{6}} | \overline{W}_3 \rangle | HH \rangle. \quad (6)$$

A similar projection on $|H\rangle$ results in

$${}_f\langle H | \Psi_6^- \rangle = -\frac{1}{\sqrt{2}} |VVVHH\rangle + \frac{1}{\sqrt{3}} | \overline{W}_3 \rangle | \Psi_2^+ \rangle - \frac{1}{\sqrt{6}} | W_3 \rangle | VV \rangle. \quad (7)$$

We have also performed such measurements related with the operator σ_z in the mode b , which has as its eigenstates $\{|H\rangle_b, |V\rangle_b\}$, while the other five photons are measured

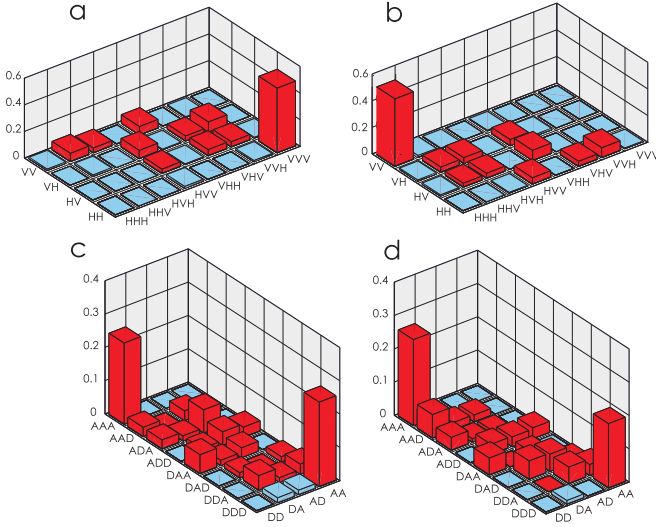


FIG. 3: Five-photon states from projective measurements. Five-fold coincidence probabilities obtained through the projection in H/V -basis of one photon. In (a) and (b) all the qubits are measured in H/V -basis and the last qubit (in the mode f) is projected onto H and V respectively. The results in (c) and (d) correspond to projective measurements in H/V -basis on the qubit b with the result H and V respectively, while the remaining five photons are measured in D/A -basis.

in the $\{|D\rangle, |A\rangle\}$ basis. The projection on $|H\rangle_b$ gives

$$\begin{aligned} {}_b\langle H | \Psi_6^- \rangle &= \frac{1}{\sqrt{2}} |GHZ_5^- \rangle + \frac{1}{\sqrt{6}} (|\Psi_2^+ \rangle + \frac{1}{\sqrt{2}} |AA \rangle) |W_3 \rangle \\ &\quad - \frac{1}{\sqrt{6}} (|\Psi_2^+ \rangle + \frac{1}{\sqrt{2}} |DD \rangle) |\bar{W}_3 \rangle, \end{aligned} \quad (8)$$

where $|GHZ_5^- \rangle = \frac{1}{\sqrt{2}} (|DDAAA \rangle - |AADDD \rangle)$ and the projection on $|V\rangle_b$ gives

$$\begin{aligned} {}_b\langle V | \Psi_6^- \rangle &= \frac{1}{\sqrt{2}} |GHZ_5^+ \rangle - \frac{1}{\sqrt{6}} (|\Psi_2^+ \rangle - \frac{1}{\sqrt{2}} |AA \rangle) |W_3 \rangle \\ &\quad - \frac{1}{\sqrt{6}} (|\Psi_2^+ \rangle - \frac{1}{\sqrt{2}} |DD \rangle) |\bar{W}_3 \rangle, \end{aligned} \quad (9)$$

where $|GHZ_5^+ \rangle = \frac{1}{\sqrt{2}} (|DDAAA \rangle + |AADDD \rangle)$.

Fig. 3 shows the results (obtained in the observation bases) for these five photon conditional polarization states. In Fig. 3.a and 3.b, we clearly see the terms $|VVVHH \rangle$, and $|HHHVV \rangle$ respectively. The terms $|DDAAA \rangle$ and $|AADDD \rangle$ are evident in both Fig. 3.c and 3.d. All these results are in agreement with theoretical predictions.

$|\Psi_6^- \rangle$ is a six-qubit entangled state, meaning that each of its qubits is entangled with all the remaining ones. In order to show that our experimental correlations reveal a six qubit entanglement we use the entanglement witness method. An entanglement witness is an observable yielding a negative value only for entangled states, the most common being the maximum overlap witness (\mathcal{W}_{max}),

which is the best witness with respect to noise tolerance [18]. The maximum overlap witness optimized for $|\Psi_6^- \rangle$ has the form

$$\mathcal{W}_{max} = \frac{2}{3} \mathbf{1}^{\otimes 6} - |\Psi_6^- \rangle \langle \Psi_6^-|, \quad (10)$$

where the factor $2/3$ is the maximum overlap of $|\Psi_6^- \rangle$ with any biseparable state. The witness detects six-partite entanglement with a noise tolerance around 34%, but it also demands a large amount of measurement settings. Since it would be an experimentally very demanding task to perform all these measurements, we have developed a reduced witness that can be implemented using only three measurement settings (this was done using the tools provided by Toth [19, 20]). Our reduced witness \mathcal{W} , is given by

$$\begin{aligned} \mathcal{W} &= \frac{77}{288} \mathbf{1}^{\otimes 6} + \frac{1}{576} \sum_{i=x,y,z} (3\sigma_i^{\otimes 2} \mathbf{1}^{\otimes 4} + 3\sigma_i \mathbf{1} \sigma_i \mathbf{1}^{\otimes 3} \\ &\quad + 3\mathbf{1} \sigma_i^{\otimes 2} \mathbf{1}^{\otimes 3} + 3\mathbf{1}^{\otimes 3} \sigma_i^{\otimes 2} \mathbf{1} + 5\sigma_i^{\otimes 2} \mathbf{1} \sigma_i^{\otimes 2} \mathbf{1} + 5\sigma_i \mathbf{1} \sigma_i^{\otimes 3} \mathbf{1} \\ &\quad + 5\mathbf{1} \sigma_i^{\otimes 4} \mathbf{1} + 3\mathbf{1}^{\otimes 3} \sigma_i \mathbf{1} \sigma_i + 5\sigma_i^{\otimes 2} \mathbf{1} \sigma_i \mathbf{1} \sigma_i + 5\sigma_i \mathbf{1} \sigma_i^{\otimes 2} \mathbf{1} \sigma_i \\ &\quad + 5\mathbf{1} \sigma_i^{\otimes 3} \mathbf{1} \sigma_i + 3\mathbf{1}^{\otimes 4} \sigma_i^{\otimes 2} + 5\sigma_i^{\otimes 2} \mathbf{1}^{\otimes 2} \sigma_i^{\otimes 2} + 5\sigma_i \mathbf{1} \sigma_i \mathbf{1} \sigma_i^{\otimes 2} \\ &\quad + 5\mathbf{1} \sigma_i^{\otimes 2} \mathbf{1} \sigma_i^{\otimes 2} + 9\mathbf{1}^{\otimes 6} - [\mathbf{1} \leftrightarrow \sigma_i]), \end{aligned} \quad (11)$$

where $[\mathbf{1} \leftrightarrow \sigma_i]$ denotes the same terms as in the sum but with $\mathbf{1}$ and σ_i interchanged. This is obtained from the maximum overlap witness as follows. First the maximum overlap witness is decomposed into direct products of Pauli and identity matrices, secondly only terms that are products of one type of Pauli matrices and identity matrices are selected (all terms that include products of at least two different Pauli matrices are deleted, remaining only non-mixed terms), e.g. $\sigma_i^{\otimes 3} \mathbf{1} \mathbf{1} \sigma_i$, $i = x, y, z$. Finally, the constant in front of $\mathbf{1}^{\otimes 6}$ in the first term of eq. (11) is chosen to be the smallest possible such that all entangled states found by the reduced witness are also found by the maximal overlap witness. Our reduced witness detects sixpartite entanglement of $|\Psi_6^- \rangle$ with a noise tolerance of 15%. The theoretical expectation value $\langle \mathcal{W} \rangle = -1/18 \approx -0.056$ and our experimental result is $\langle \mathcal{W} \rangle = -0.023 \pm 0.012$, showing entanglement with 2.0 standard deviations.

In summary, we have experimentally tested the property of rotational invariance of the six-photon state produced by our setup. The state is indeed entangled, and various different entangled states can be obtained out of it with the use of projective measurements of one of the qubits. We would like to note that the interference contrast is high enough for our setup to be used in demonstrations of various six-party quantum informational applications (quantum reduction of communication complexity of some joint computational tasks, secret sharing, etc.).

Acknowledgements This work was supported by Swedish Research Council (VR). M.Ž. was supported by

Wenner Gren Foundations and by the EU programme QAP (Qubit Applications, No. 015858).

-
- [1] J.-W. Pan, Z.-B. Chen, M. Żukowski, H. Weinfurter, and A. Zeilinger, A., Preprint available at quant-ph/0805.2853 (2008).
 - [2] W. H. Zurek, Phys. Today **44** (no. 10), 36 (1991).
 - [3] P. Zanardi and M. Rasetti, Phys. Rev. Lett. **79**, 3306 (1997).
 - [4] D. A. Lidar, I. L. Chuang, and K. B. Whaley, Phys. Rev. Lett. **81**, 2594 (1998).
 - [5] P. G. Kwiat, *et al.*, Science **290**, 498 (2000).
 - [6] L. Viola, *et al.*, Science **293**, 2059 (2001).
 - [7] M. Bourennane, *et al.*, Phys. Rev. Lett. **92**, 107901 (2004).
 - [8] A. Cabello, Phys. Rev. A **75** 020301 (2007).
 - [9] S. D. Bartlett, T. Rudolph, and R.W. Spekkens, Phys. Rev. Lett. **91**, 027901 (2003).
 - [10] M. Hillery, V. Bužek, and A. Berthiaume, Phys. Rev. A **59**, 1829 (1999).
 - [11] S. Gaertner, M. Bourennane, Ch. Kurtsiefer, and H. Weinfurter, Phys. Rev. Lett. **98**, 020503 (2007).
 - [12] C.-Y. Lu, *et al.*, Nature Physics, **3**, 91 (2007).
 - [13] A. Zeilinger, M. A. Horne, H. Weinfurter, and M. Żukowski, Phys. Rev. Lett., **78**, 3031 (1997).
 - [14] M. Żukowski, A. Zeilinger, and H. Weinfurter, Ann. N. Y. Acad. Sci., **755**, 91 (1995).
 - [15] M. Rådmark, M. Wieśniak, M. Żukowski, M. Bourennane, e-print arXiv:0903.2454.
 - [16] H. Weinfurter and M. Żukowski, Phys. Rev. A **64**, 010102(R) (2001).
 - [17] S. Gaertner, M. Bourennane, M. Eibl, Ch. Kurtsiefer, and H. Weinfurter, Appl. Phys. B **77**, 803 (2003).
 - [18] M. Bourennane, *et al.*, Phys. Rev. Lett. **92**, 087902 (2004).
 - [19] G. Tóth and O. Gühne, Phys. Rev. Lett. **95**, 120405 (2005).
 - [20] G. Tóth, Preprint available at quant-ph/07090948, (2007).